PARTICLE-IN-CELL SIMULATIONS OF ALFVÉN-CYCLOTRON WAVE SCATTERING: PROTON VELOCITY DISTRIBUTIONS

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Abstract

Alfvén-cyclotron fluctuations propagating parallel or antiparallel to the background magnetic field $\mathbf{B}_o$ help shape solar wind ion velocity distributions $f_i(\mathbf{v})$. Alfvén waves may be generated at low, nonresonant frequencies and, by propagation through the inhomogeneous plasma, attain ion cyclotron resonances and thereby scatter the $f_i(\mathbf{v})$ to anisotropy. Ion anisotropies of sufficient magnitude lead to the growth of ion cyclotron instabilities; the resulting enhanced Alfvén-cyclotron fluctuations scatter ions so as to reduce the anisotropy. Here particle-in-cell simulations were carried out in a magnetized, homogeneous, collisionless plasma of electrons and one species of ions to study the evolution of the $f_i(\mathbf{v})$ in response to both of these scattering processes. A simulation with a spectrum of right-traveling Alfvén-cyclotron fluctuations imposed at $t = 0$ leads to non-Maxwellian ion distributions. The computations show that the pitch-angle scattering of left-traveling ($v_\parallel < 0$) ions becomes weaker as $v_\parallel$ becomes less negative, but also that this scattering can transport ions across the condition $v_\parallel = 0$, where the subscript denotes the direction parallel to $\mathbf{B}_o$.

1. Introduction

There have been many measurements of ion anisotropies such that $T_\perp > T_\parallel$ in both the solar corona and the solar wind (Here and throughout this manuscript $\perp$ and $\parallel$ denote directions relative to the background magnetic field $\mathbf{B}_o$.). Ion anisotropies in the corona inferred from remote sensing observations were carefully reviewed by *Hollweg and Isenberg* [2002]. Solar wind *in situ* measurements of the proton anisotropy $T_{\perp p}/T_{\parallel p} > 1$ have been reported primarily in the fast wind and include *Bame et al.*, 1975; *Marsch et al.*, 1982a; *Feldman et al.*, 1996 and *Neugebauer et al.*, 2001.

Collisionless wave-particle scattering by Alfvén-cyclotron fluctuations is a possible explanation for these anisotropies. Although there are no direct observations of Alfvén waves in the corona, large-amplitude, low frequency Alfvénic fluctuations are observed to be ubiquitous in the solar wind [*Coleman*, 1968; *Goldstein et al.*, 1994]. In particular, *Marsch and Tu* [2001] and *Tu and Marsch* [2002] have demonstrated proton velocity distributions observed from the Helios spacecraft which exhibit anisotropies consistent with pitch-angle scattering by Alfvén-cyclotron waves propagating away from the Sun.

A substantial body of theory has addressed the wave-particle interactions between Alfvén fluctuations and ions in the solar corona and/or the solar wind [*Marsch et al.*, 1982b; *Isenberg and Hollweg*, 1983; *Hollweg and Johnson*, 1988; *McKenzie*, 1994; *Tam and Chang*, 1999; *Li et al.*, 1999; *Hollweg*, 1999a, 1999b, 2000; *Cranmer*, 2000, 2001; *Tu and Marsch*, 2001; *Isenberg et al.*, 2001; *Isenberg*, 2001, 2003; *Vocks and Marsch*, 2001, 2002; *Vocks*, 2002; *Gary et al.*, 2001a]. The common scenario for many of these theories is that low frequency, nonresonant Alfvénic fluctuations are generated at the coronal base and perhaps throughout the corona [*Cranmer et al.*, 1999]; as they propagate upward, the decreasing magnetic field leads to wave-particle interactions at cyclotron resonances of successively larger charge-to-mass ratio ions. The resulting pitch-angle scattering drives the resonant ions to larger values of $v_\perp$, thereby presenting an explanation for the observations of $T_{\perp i} > T_{\parallel i}$.

Whatever the source, if the $T_\perp/T_\parallel$ of any ion species becomes sufficiently large, it will excite an electromagnetic ion cyclotron anisotropy instability. It is well established, through both solar wind observations [*Gary et al.*, 2001b, 2002, 2003] and simulations [*Gary et al.*, 1997, 2000, 2001a; *Ofman et al.*, 2001], that the resulting short wavelength, enhanced fluctuating fields scatter the driving ions and reduce or at least constrain the driving ion anisotropies. We here examine the effects on the ion velocity distribution $f_i(\mathbf{v})$ of wave-particle scattering by both an imposed spectrum of Alfvén-cyclotron fluctuations and by the spontaneously growing waves of the ion cyclotron instability.

We define the average parallel flow speed of the jth species as

$$v_{oj} \equiv \frac{1}{n_j} \int d^3v \; v_\parallel \; f_j(\mathbf{v}).$$

We also define two kinetic energy densities of the jth species:

$$W_{\parallel j} \equiv \frac{m_j}{2} \int d^3v \; v_\parallel^2 \; f_j(\mathbf{v})$$

and

$$W_{\perp j} \equiv \frac{m_j}{4} \int d^3v \; v_\perp^2 \; f_j(\mathbf{v})$$

If the velocity distribution becomes bi-Maxwellian, then $T_{\parallel j} = 2W_{\parallel j}/n_j$ and $T_{\perp j} = 2W_{\perp j}/n_j$. Other jth species quantities used here include $\beta_{\parallel j} \equiv 8\pi n_j k_B T_{\parallel j}/B_o^2$; the plasma frequency, $\omega_j \equiv \sqrt{4\pi n_j e_j^2/m_j}$; the cyclotron frequency, $\Omega_j \equiv e_j B_o/m_j c$; and the thermal speed, $v_j \equiv \sqrt{k_B T_{\parallel j}/m_j}$. The Alfvén speed is $v_A \equiv B_o/\sqrt{4\pi n_e m_p}$.

2. Simulation Code

We used a two-and-one-half-dimensional (spatial variations in x and y, velocity variations in all three components) particle-in-cell electromagnetic simulation code [*Buneman*, 1993] to compute the results described in the following sections. We consider a collisionless, magnetized, homogeneous plasma and choose $\mathbf{B}_o = \hat{\mathbf{x}} B_o$. Our concern here is scattering by fluctuations which propagate at $\mathbf{k} \times \mathbf{B}_o = 0$, so we use a configuration in which the system dimensions are $L_x = 760\ \Delta$ and $L_y = 10\ \Delta$, where Δ is the cell size. The particle number density for each species is about 100 per cell, and the integration time step is $\Omega_i \Delta t = 1.4 \times 10^{-3}$. Periodic boundary conditions are imposed on both particle and field quantities in both the x- and y-directions. For both of our simulations, the electrons are initially Maxwellian, the initial electron temperature is equal to the ion temperature parallel to $\mathbf{B}_o$, the initial ion beta is $\beta_{\parallel i} = 0.05$, the ion skin depth is $c/\omega_i = 40\Delta$, and the ion Debye length is $v_i/\omega_i = \Delta$. The ion to electron mass ratio is $m_i/m_e = 16$.

Although we use an artificially small ion/electron mass ratio in our simulations, we subsequently use the term "proton" (subscript p) to refer to the ions, implying that we are studying only the majority ion species. Of course, there is much additional physics involved in the wave-particle interactions of Alfvén-cyclotron fluctuations and instabilities with minor, heavy-ion species of the corona and solar wind [e.g., see the recent simulations of *Liewer et al.*, 2001; *Ofman et al.*, 2002; *Gary et al.*, 2003], but such a study is beyond the purview of the present manuscript. At the minimum, our model of a single ion species should apply to protons in the corona and solar wind whenever the heavy ions are nonresonant. This condition arises, for example, when heavy-ion/proton relative flow speeds v_{ip} satisfy $v_{ip} \gtrsim 0.5 v_A$. Under this condition, the exchange of energy between Alfvén-cyclotron fluctuations and the heavy ions becomes weak. This is true whether the fluctuations arise locally from the electromagnetic proton cyclotron anisotropy instability (hereafter simply the "proton cyclotron instability") [e.g., Fig. 7 of *Gary et al.*, 2003], or are imposed from some nonlocal source [e.g., Fig. 5 of *Gary et al.*, 2001c].

3. Proton Cyclotron Instability

This section describes the evolution of the proton velocity distribution under the influence of enhanced field fluctuations generated by the proton cyclotron instability driven by a sufficiently large $T_{\perp p}/T_{\parallel p} > 1$. Its properties as derived from linear theory in a plasma with protons as the only ions are summarized in *Gary* [1993]; the growing modes propagate with left-hand polarization, with maximum growth rate at $\mathbf{k} \times \mathbf{B}_o = 0$ and at frequencies somewhat below the proton cyclotron frequency. If, as we assume here, $f_p(\mathbf{v})$

is gyrotropic and symmetric in $v_\parallel$ [that is, $f_p(v_\parallel, v_\perp) = f_p(-v_\parallel, v_\perp)$], the enhanced fluctuations from instability growth also exhibit a right-left symmetry. That is, fluctuations propagating parallel to $\mathbf{B}_o$ ($\omega_r/k_\parallel > 0$), will have the same growth rate $\gamma(k_\parallel)$ and polarization as fluctuations propagating antiparallel to the background magnetic field ($\omega_r/k_\parallel < 0$). These symmetry properties imply that the consequences of wave-particle scattering by the resulting enhanced fluctuations should preserve the right-left symmetry of the velocity distribution, and our simulations indeed demonstrate that this is the case.

Beyond this expected symmetry, however, we ask a more detailed question: If we begin our simulations with a bi-Maxwellian $f_p(\mathbf{v})$, will its bi-Maxwellian character be preserved as the fluctuations become enhanced and the temperature anisotropy $T_{\perp p}/T_{\parallel p}$ is reduced? Our previous analyses of the proton cyclotron instability indicated a positive answer to this question [*Gary et al.*, 1996, 1997]. However, those analyses were based primarily on reduced velocity distributions, which are one-dimensional in character. Pitch-angle scattering by Alfvénic fluctuations involves an exchange of charged particle velocities between $v_\parallel$ and $v_\perp$, which is a two-dimensional velocity space process. Thus it is appropriate to re-examine this question using two-dimensional diagnostics.

We carried out a simulation of the proton cyclotron instability using the parameters described in Section 2, including the initial value of $\beta_{\parallel p} = 0.05$. The initial proton velocity distribution was taken to be a bi-Maxwellian with an initial temperature anisotropy $T_{\perp p}/T_{\parallel p} = 10.3$. Under such initial conditions, linear Vlasov theory predicts that the proton cyclotron instability arises with maximum growth rate $\gamma_m/\Omega_p = 0.20$. With the given simulation box size of $L_x\omega_p/c = 19$, linear theory also predicts that five modes should have growth rates $\gamma/\Omega_p > 0.10$, implying that there should be a broad spectrum of enhanced Alfvén-cyclotron fluctuations to scatter the protons.

Figure 1 shows (a) the total fluctuating magnetic field energy density and (b) the associated proton temperature anisotropy, both as functions of time, for this simulation. The results are the same as for hybrid simulations of this growing mode [*Gary et al*, 2000]; the fluctuating magnetic fields grow rapidly to saturation, and $T_{\perp p}/T_{\parallel p}$ is rapidly reduced to a value which corresponds to an instability threshold of weak growth.

Figure 2 shows contours of constant phase space for $f_p(v_\parallel, v_y)$ at four times: $\Omega_p t = 0$, 22.4 (midway through the exponential growth of the instability), 40 (near saturation of the fluctuating fields), and 56 (the end of the simulation). During instability growth the distribution is clearly distorted from its initial bi-Maxwellian shape; the cyclotron resonant particles at relatively large $v_\perp$ have just begun their pitch-angle migration to larger values of $v_\parallel$. But by the time of saturation, the response of the particles scattered by the fastest growing waves is essentially complete and the distribution has returned to a bi-Maxwellian-like shape. At times well after saturation, the scattering continues weakly, but the bi-Maxwellian shape is approximately maintained.

Figure 3 presents two comparisons of velocity distributions which allow a more quantitative analysis of the scattered $f_p(\mathbf{v})$. Figure 3(a) illustrates the difference between the $f_p(\mathbf{v})$ at late simulation time and a theoretical bi-Maxwellian distribution where $T_{\perp p}/T_{\parallel p} = 3.2$ is chosen to yield the best match with the simulation result. The difference is primarily statistical noise, confirming that the simulated distribution is close to a bi-Maxwellian over the full range of thermal speeds. We regard this result as confirming our earlier conclusion that scattering by a broad spectrum of enhanced fluctuations from the proton cyclotron instability will retain the bi-Maxwellian character of an anisotropic velocity distribution, even as that scattering acts to reduce that anisotropy.

Panel (b) shows how that anisotropy is reduced, illustrating the difference between $f_p(v_\parallel, v_y)$ at $\Omega_p t = 56$ and $f_p(v_\parallel, v_y)$ at $t = 0$. This shows that scattering by enhanced fluctuations from the instability moves particles from smaller $v_\parallel$ to larger $v_\parallel$, exactly the response expected for pitch-angle scattered protons. This figure also shows that the proton response is indeed symmetric in $v_\parallel = 0$, just what we expect for scattering by an ensemble of fluctuations which propagate both parallel and antiparallel to $\mathbf{B}_o$.

4. Imposed Fluctuations

This section describes results from our study of the proton velocity distribution as it evolves in a simulation in which an electromagnetic fluctuation spectrum at $\mathbf{k} \times \mathbf{B}_o = 0$ is imposed at $t = 0$. The initial magnetic field fluctuation spectrum is

$$\delta\mathbf{B}(x, t = 0) = \sum_{n=1}^{5} \hat{\mathbf{y}}\ \delta B_n \sin(-k_n x + \phi_n) + \hat{\mathbf{z}}\ \delta B_n \cos\,(-k_n x + \phi_n) \tag{1}$$

with $k = k_\parallel$ understood throughout this section and mode parameters as stated in Table 1. The last column of this table states the complex frequency $\omega(k) = \omega_r + i\gamma$ which is obtained from the solution of the linear Vlasov dispersion equation [e.g., *Gary*, 1993] for left-hand polarized electromagnetic fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$ and $\beta_{\parallel p} = 0.05$. The spatially homogeneous part of the initial proton and electron velocity distributions in the simulation are isotropic and Maxwellian. A spatially inhomogeneous part is added to the initial proton velocity distribution to assure that it is self-consistent with Equation (1).

Our application of magnetic fluctuations emulates the procedure of *Liewer et al.* [2001], who imposed an initial spectrum of Alfvén waves on a spatially homogeneous simulation, rather than that of *Ofman et al.* [2002], who drove a magnetic power spectrum in a small region of the computational domain throughout the duration of their simulations. We believe that the former procedure is a more realistic representation of solar wind conditions, for the following reason. Under the scenario outlined in the Introduction, there are two processes involved in the transfer of wave energy to solar wind ions. First is the migration of magnetic fluctuation energy from long, nonresonant wavelengths to shorter,

Table 1. Initial Fluctuation Spectrum: Parameters

n	δB_n	$k_n c/\omega_p$	ϕ_n	ω(Alfvén)$/\Omega_p$
1	0.0357	1.00	0.0	0.560 $-i$0.014
2	0.0357	1.25	1.11π	0.594 $-i$0.057
3	0.0357	1.50	0.88π	0.619 $-i$0.119
4	0.0357	1.75	0.66π	0.640 $-i$0.192
5	0.0357	2.00	1.11π	0.658 $-i$0.273

cyclotron resonant wavelengths. Second is the absorption of wave energy by cyclotron damping on the ions. The former process proceeds, on average, at about the solar wind expansion rate. The latter process proceeds at a variable rate; the faster wave energy is pumped to the cyclotron resonant wavenumbers, the deeper that spectrum penetrates to shorter wavelengths and, as is clear from Figure A.1, the faster the damping. Thus in steady state, the overall rate of energy flow is determined by the slower rate, that is, the migration of energy in wavenumber. Thus the rate of fluctuating field energy flow to the cyclotron resonance must be orders of magnitude smaller than the damping rates illustrated in Table 1, and any simulation of an imposed magnetic fluctuation spectrum should not replenish that wave energy on any time scale faster than about $10^4/\Omega_p$.

All modes of Equation (1) have negative helicity; that is, they have a right-hand sense of rotation in the direction of $\mathbf{k}$. This means that they may excite both left-hand polarized Alfvén-cyclotron waves which propagate parallel to $\mathbf{B}_o$ (i.e., with $\omega_r > 0$) as well as right-hand polarized magnetosonic/whistler waves which propagate antiparallel to the background magnetic field with $\omega_r < 0$. Linear Vlasov dispersion theory predicts that, for the wavenumbers of Table 1, the right-hand polarized waves all correspond to $|\omega_r| > \Omega_p$ and are undamped at the low β_p used here, so that they should not contribute to resonant wave-particle interactions in this simulation. In contrast, linear Vlasov theory predicts that the left-hand polarized fluctuations are cyclotron resonant with the protons, and have substantial damping as indicated by Table 1.

Figure 4(a) shows the time history of the fluctuating magnetic field energy density from this computation. During the first proton cyclotron period ($0 \leq \Omega_p t \lesssim 2\pi$), there is relatively rapid damping of the fluctuations, after which the fluctuating field energy settles down to a relatively constant value for the remainder of the simulation. Panel (b) illustrates the time history of the parallel and perpendicular proton kinetic energies for this simulation. There is a relatively modest increase in $W_{\perp p}$ and a corresponding modest decrease in $W_{\parallel p}$ during the time of magnetic field energy decrease. This is the response we expect for the protons subject to a spectrum of Alfvén-cyclotron fluctuations. At the time of maximum scattering, $W_{\perp p}/W_{\parallel p} = 2.1$; if the distributions were approximately bi-Maxwellian, the corresponding temperature anisotropy would not be sufficiently large to excite the proton cyclotron instability. We have also calculated the average proton parallel flow speed v_{op} as a function of time. There is no appreciable change in this quantity: -0.001 $\lesssim v_{op}/v_p < 0.002$ throughout the simulation.

Figure 5 illustrates the proton velocity distribution at $\Omega_p t = 0$, 5.6, 11.2, and 16.8; the scattered $f_p(\mathbf{v})$ are clearly not bi-Maxwellian. Pitch-angle scattering of the protons at $v_\parallel < 0$ by the right-traveling Alfvén-cyclotron waves is evident, but the absence of any clear increase of the distribution width at $v_\parallel > 0$ again indicates that the proton cyclotron instability has not been excited in this case.

Figure 6 illustrates the proton velocity distribution at $\Omega_p t = 56.0$, with the phase speeds of the five modes from Table 1 indicated by dots on the right-hand side of the illustration. The decreasing values of ω_r/k with increasing kc/ω_p correspond to the increasing dispersion of the modes as they encounter a successively stronger proton cyclotron resonance (see Figure A.1). The dots on the left-hand side of the figure represent $v_\parallel$ solutions of Equation (A.1) with $\omega = \omega_r$ for the same five modes of Table 1. If, as is assumed by some theories, contours of constant phase space density should develop along the trajectories of pitch-angle scattered ions, we should find a correspondence between the two. Using the prescription of *Isenberg and Lee* [1996], the Appendix describes our calculation of such trajectories for the spectrum of Table 1. The thick solid lines represent trajectories which are solutions of Equation (A.6); that is, they show the velocity space paths of protons which are pitch-angle scattered by the succession of waves stated in Table 1.

There is a good match between the solid lines and the simulated contours of constant phase space at relatively small $v_\perp$. This agreement indicates that pitch-angle scattering is complete over a significant range of these trajectories. However, the simulated distribution contours fall away from the theoretical curves as $v_\parallel$ becomes less negative, implying that the pitch-angle scattering becomes weaker as $v_\parallel$ approaches zero. This supports the hypothesis that pitch-angle scattering is the primary consequence of wave-particle interactions in our simulation.

There is another important difference between our simulated distributions and the proton velocity distributions assumed in the ion heating theory of *Isenberg et al.* [2001] and *Isenberg* [2001; 2003]. In that theory, Equation (A.2) with $\omega = \omega_r$ is taken to be the proton cyclotron resonance condition, so that if k is assumed positive, then the condition $v_\parallel = 0$ represents the boundary between two distinct regimes. That is, for electromagnetic cyclotron fluctuations with $\omega_r < \Omega_p$, this condition separates the protons at $v_\parallel < 0$ which resonate with fluctuations at $\omega_r/k > 0$ from the protons at $v_\parallel > 0$ which resonate with fluctuations at $\omega_r/k < 0$. The results of the theory then depend critically on how ions are transported across the $v_\parallel = 0$ plane (Compare *Isenberg* [2001] with *Isenberg* [2003]). The $v_\parallel = 0$ condition plays a similar, important role in the ion cyclotron heating theory of *Hollweg* [1999b, 2000].

Our simulated $f_p(\mathbf{v})$ do not indicate any such radical change in the physics at $v_\parallel = 0$. A comparison of our Figure 6 with, for example, Fig. 2 of *Isenberg* [2001] shows very similar shapes at negative parallel velocities, but major differences at $v_\parallel \geq 0$. In particular our contours of constant phase space density are not only continuous across $v_\parallel = 0$ but have continuous derivatives across this transition. The observed distributions of *Marsch and Tu* [2001] and *Tu and Marsch* [2002], as well as all observed velocity distributions of which we are aware, exhibit the same properties. Figure 6 suggests that the pitch-angle scattering which shapes the left-hand side of the $f_p(\mathbf{v})$ also carries a limited number of particles to the right-hand side, giving rise to the peculiar "pips" ("PItch-angle ProtuberanceS") on the distribution at $0 < v_\parallel/v_p << 1$ and $1 < v_\perp/v_p$.

How can this violation of a fundamental assumption of the Isenberg and Hollweg theories occur? The answer rests with the context of Equation (A.1); in kinetic theory, ω is a complex variable with a non-zero imaginary part, especially for waves which are cyclotron resonant. In the linear and weakly nonlinear Vlasov theories of electromagnetic fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$, the effectiveness of the species j cyclotron resonance is derived from a velocity space integral which includes a contribution which we call the "resonance factor"

$$R_j(v_\parallel) \equiv \frac{\Omega_j}{kv_\parallel - \omega + \Omega_j} \qquad (2)$$

[See, for example, the linear conductivity of Eq. (5.1.14) in *Gary*, 1993]. The imaginary part of this factor determines the part of $f_p(v_\parallel)$ which contributes to both linear damping of the waves and second-order scattering of those particles [e.g., *Gary and Tokar*, 1984], including the pitch-angle scattering considered here.

In the limit of zero damping, the imaginary part of $R_j(v_\parallel)$ becomes a delta function. In this case the fundamental assumption of *Isenberg* [2001, 2003] that ions are not pitch-angle scattered across $v_\parallel = 0$ is valid. However, at zero damping there is no transfer of energy from the waves to the particles, and pitch-angle scattering of thermal particles is very weak or negligible. Only if $\gamma \neq 0$ is there significant wave-particle energy transfer, but in this case the resonance factor has a non-zero width which increases as $|\gamma|$ becomes larger. It is precisely the finite width of $R_j(v_\parallel)$ which allows some particles to be pitch-angle scattered to $v_\parallel > 0$ and to create the "pips" illustrated in Figures 5 and 6.

Figure 7 illustrates the resonance factor for each of the five modes of Table 1, showing how this factor becomes broader with increasing wavenumber and increasing damping rate. The gradual decrease in the amplitudes of these R_j as $v_\parallel$ changes from negative to positive values further supports our twin conclusions that pitch-angle scattering becomes weaker as $v_\parallel$ becomes less negative but that this scattering can nevertheless still transport ions across $v_\parallel = 0$.

5. Conclusions

We have used particle-in-cell computer simulations of collisionless plasmas to study the shaping of proton velocity distributions by wave-particle interactions with Alfvén-cyclotron fluctuations. Our first simulation confirmed that wave-particle scattering by a broad spectrum of waves from the proton cyclotron instability maintains the initially bi-Maxwellian character of the proton velocity distribution. Our second simulation demonstrated the non-Maxwellian properties of $f_p(\mathbf{v})$ which arise from pitch-angle scattering by an imposed spectrum of Alfvén-cyclotron fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$. Two important conclusions from the latter simulation are, first, that pitch-angle scattering becomes weaker as $v_\parallel$ becomes less negative and, second, that this scattering can move protons across the $v_\parallel = 0$ condition. Inclusion of these results into the theory of *Isenberg* [2003] might lead to changes in the conclusions of that paper.

Our results suggest several topics which might be addressed by extensions of these computations. First, the spectrum of Alfvén-cyclotron fluctuations initially imposed on the system should be varied, using different choices of initial modes and initial wave amplitudes, as in *Ofman et al.* [2002]. One goal of this exercise should be to determine the modes which are most effective in scattering protons; long wavelength modes are nonresonant, and short wavelength modes are very strongly damped, so we expect some intermediate value of kc/ω_p to be most efficient. Another goal of this exercise should be to find the wave amplitudes necessary to excite unstable modes, whether these modes are driven by the overall $T_{\perp p} > T_{\parallel p}$ or by the local (in velocity space) pips, and what are the velocity space consequences of scattering by these instabilities. Second, the initial waves imposed on the system could be generalized to include magnetic fluctuations propagating oblique to $\mathbf{B}_o$; in these cases the Landau resonance at $v_\parallel = \omega_r/k_\parallel$ becomes effective, and both electrons and protons can participate in the wave-particle interactions. Third, the initial particle velocity distributions could be generalized to include heavy ions such as alpha particles as a minority species. Such a computation could be used to test the predictions that a spectrum of Alfvén fluctuations at long wavelengths will cyclotron damp upon and thereby preferentially heat the alphas if the alpha/proton relative flow is relatively small, but that if the alpha/proton relative flow becomes an appreciable fraction of v_A, the alpha cyclotron damping becomes weak, and proton cyclotron damping at shorter wavelengths becomes the dominant wave-particle process. Finally, an ensemble of magnetosonic/whistler fluctuations propagating at $\mathbf{k} \times \mathbf{B}_o = 0$ could be imposed on the system; linear theory [*Gary*, 1999; *Stawicki et al.*, 2001] predicts that such waves are subject to appreciable proton cyclotron damping at $\beta_p \gtrsim 2.5$. We expect that such damping will also drive a proton anisotropy of the type

$T_{\perp p} > T_{\| p}$, although the velocity space details may be different from that due to the Alfvén-cyclotron fluctuations illustrated here. Except for the second, each of these activities address wave-ion interactions and could be executed through the use of hybrid simulations which represent electrons as a fluid, rather than the full particle-in-cell simulations used here.

Appendix

If a charged particle is pitch-angle scattered by an electromagnetic fluctuation at $\mathbf{k} \times \mathbf{B}_o = 0$ with complex frequency ω and $\mathbf{k} = \hat{\mathbf{B}}_o k$, then the trajectory of that particle should satisfy both the cyclotron resonant condition

$$\omega - k v_{\|} = +\Omega_p \tag{A.1}$$

and the condition of kinetic energy conservation in the wave frame

$$(v_{\|} - \omega_r/k)^2 + v_{\perp}^2 = \text{constant} \tag{A.2}$$

If the pitch-angle scattering is due to a spectrum of waves with constant phase speed, say, v_A, then the $(v_{\|}, v_{\perp})$ trajectory of such a particle follows immediately from Equation (A.2). This case, however, has limited applicability to the scattering of thermal particles, because the cyclotron resonances for such particles typically lie in strongly dispersive and damped regimes, as illustrated in Figure A.1.

A procedure for calculating the $(v_{\|}, v_{\perp})$ trajectory for a charged particle pitch-angle scattered by a spectrum of waves with phase speeds which vary with wavenumber has been given by *Isenberg and Lee* [1996]. The appropriate equation is

$$v_{\|}^2 + v_{\perp}^2 - 2 \int^{v_{\|}} \frac{\omega_r(v'_{\|})}{k(v'_{\|})} dv'_{\|} = \text{constant} \tag{A.3}$$

where the $v_{\|}$ dependence of the frequency and wavenumber under the integral are determined by the simultaneous solution of Equation (A.1) and the dispersion equation for $\omega_r(k)$. *Isenberg and Lee* [1996], as well as *Isenberg* [2003], assume a cold plasma dispersion equation, enabling a closed-form analytic solution of Equation (A.3). Here we pursue a somewhat different approach to the solution of (A.3).

We begin with the fully thermal dispersion equation solution for Alfvén-cyclotron fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$ in a collisionless plasma. For Maxwellian protons and electrons, the exact numerical result at $\beta_p = 0.05$ is illustrated in Figure A.1, where the five vertical arrows indicate the five modes applied as initial conditions to the simulation described in Section 4. For the wavenumber range of these modes, $1.0 \lesssim kc/\omega_p \lesssim 2.0$, the mode is highly dispersive; if we assume a solution of the form

$$\frac{\omega_r}{\Omega_p} = \alpha_1 + \alpha_2 \frac{kc}{\omega_p} \tag{A.4}$$

a least-squares fit to this k-range yields $\alpha_1 = 0.47$ and $\alpha_2 = 0.09$. The reader will note that this does not yield conventional Alfvén dispersion in the long-wavelength limit; our response is that we are not concerned with that limit because those fluctuations yield only weak scattering of the thermal particles.

We combine Equation (A.1) with $\omega = \omega_r$ and Equation (A.4) to obtain

$$\frac{k(v_\parallel)c}{\omega_p} = \frac{(1-\alpha_1)}{(\alpha_2 - v_\parallel/v_A)} \tag{A.5}$$

We substitute the frequency of Equation (A.4) and the wavenumber of Equation (A.5) into Equation (A.3) and carry out the integral to obtain

$$\frac{v_\perp^2}{v_A^2} + \frac{1}{(1-\alpha_1)}\frac{v_\parallel^2}{v_A^2} - \frac{2\alpha_2}{(1-\alpha_1)}\frac{v_\parallel}{v_A} = \text{constant} \tag{A.6}$$

Solutions of this equation for $\alpha_1 = 0.47$ and $\alpha_2 = 0.09$ and different values of the constant are plotted as the heavy solid lines in Figure 6.

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Figure Captions

Figure 1. Results from the simulation of the proton cyclotron instability described in Section 3. Panel (a) shows the fluctuating magnetic field energy density, and panel (b) illustrates the proton temperature anisotropy, both as functions of time.

Figure 2. Proton velocity distributions from the simulation of the proton cyclotron instability described in Section 3. The four panels represent $f_p(v_\|, v_y)$ summed over all x at the four stated times.

Figure 3. Results from the simulation of the proton cyclotron instability described in Section 3. Panel (a) shows the difference between $f_p(v_\|, v_y)$ at $\Omega_p t = 56$ and a bi-Maxwellian velocity distribution with $T_{\perp p}/T_\|$ =3.2. Panel (b) shows the difference between $f_p(v_\|, v_y)$ at $\Omega_p t = 56$ and $f_p(v_\|, v_y)$ at $t = 0$.

Figure 4. Results from the simulation described in Section 4, in which the magnetic fluctuation spectrum of Equation (1) with parameters as given in Table 1 has been imposed

at $t = 0$. Panel (a) illustrates the time history of the fluctuating magnetic field energy density normalized to the initial value of that quantity. Panel (b) shows the time history of the dimensionless $W_{\|p}$ and $W_{\perp p}$.

Figure 5. Proton velocity distributions from the simulation described in Section 4. The four panels represent $f_p(v_\|, v_y)$ summed over all x at the four stated times. There are no major changes in the $f_p(v_\|, v_y)$ at subsequent times.

Figure 6. The proton velocity distribution $f_p(v_\|, v_y)$ at $\Omega_p t = 56.0$ from the simulation described in Section 4. The phase speed $(\omega_r/k_\|)$ of the five left-hand polarized modes given in the right-hand column of Table 1 are indicated by five dots at $v_\perp = 0$ and $v_\| > 0$. The five dots at $v_\perp = 0$ and $v_\| < 0$ represent the corresponding cyclotron resonant velocities, that is, solutions of Equation (A.1) with $\omega = \omega_r$ and $k_\|$ as given in Table 1. The five corresponding solutions of Equation (A.6) are indicated by solid heavy lines.

Figure 7. The proton resonance factors of Equation (2) for the five modes of Table 1 as functions of the proton velocity parallel to $\mathbf{B}_o$.

Figure A.1. The real frequency (solid line) and damping rate (dotted line) as functions of the wavenumber for the Alfvén-cyclotron wave at $\beta_p = 0.05$. The dashed line represents the real frequency of the dispersionless Alfvén wave $\omega_r = kv_A$. The five vertical arrows point to the frequencies of the five modes given in Table 1.